

# LONG MEMORY ESTIMATION OF STOCHASTIC VOLATILITY FOR INDEX PRICES

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Dedicated to my beloved family and friends

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## ABSTRACT

One of the typical ways of measuring risk associated with persistence in financial data set can be done through studies of long memory and volatility. Finance is a branch of economics concerned with resource allocation which deals with money, time and risk and their interrelation. The investors invest at risk over a period of time for the opportunity to gain profit. Since the last decade, the complex issues of long memory and short memory confounded with occasional structural break had received extensive attention. Structural breaks in time series can generate a strong persistence and showing a slower rate of decay in the autocorrelation function which is an observed behaviour of a long memory process. Besides that, the persistence in volatility cannot be captured easily because some of the mathematical models are not able to detect these properties. To overcome these drawbacks, this study developed a procedure to construct long memory stochastic volatility (LMSV) model by using fractional Ornstein-Uhlenbeck (fOU) process in financial time series to evaluate the degree of the persistence property of the data. The drift and volatility parameters of the fractional Ornstein-Uhlenbeck model are estimated separately using least square estimator (LSE) and quadratic generalized variations (QGV) method respectively. Whereas, the long memory parameter namely Hurst parameter is estimated by using several heuristic methods and a semi-parametric method. The procedure of constructing LMSV model and the estimation methods are applied to the real daily index prices of FTSE Bursa Malaysia KLCI over a period of 20 years. The findings showed that the volatility of the index prices exhibit long memory process but the returns of the index prices do not show strong persistence properties. The root mean square errors (RMSE) obtained from various methods indicates that the performances of the model and estimators in describing returns of the index prices are good.

## ABSTRAK

Salah satu cara tipikal untuk mengukur risiko yang berkaitan dengan keterusan dalam set data kewangan boleh dilakukan melalui kajian memori panjang dan turun naik. Kewangan adalah satu cabang ekonomi berkenaan dengan peruntukan sumber yang berkaitan dengan wang, masa dan risiko dan saling kaitan antara mereka. Para pelabur melabur dalam keadaan risiko sepanjang tempoh masa untuk peluang bagi mendapatkan keuntungan. Sejak sedekad yang lalu, isu-isu yang kompleks memori panjang dan memori pendek yang dikaburi dengan pemisahan struktur telah mendapat perhatian luas. Pemisahan struktur dalam siri masa boleh menjana keterusan yang kuat dan menunjukkan kadar susutan yang lebih perlahan dalam fungsi autokorelasi adalah satu tingkah laku yang diperhatikan dalam proses memori yang panjang. Selain itu, keterusan dalam turun naik tidak boleh diperhati dengan mudah kerana beberapa model matematik tidak dapat mengesan sifat-sifat ini. Untuk mengatasi kelemahan ini, kajian ini membina prosedur untuk membangunkan model memori panjang turun naik stokastik (LMSV) dengan menggunakan proses pecahan Ornstein-Uhlenbeck (fOU) dalam siri masa kewangan untuk menilai tahap sifat keterusan data. Parameter hanyutan dan turun naik model pecahan Ornstein-Uhlenbeck dianggarkan secara berasingan masing-masing menggunakan kaedah kuasa dua terkecil (LSE) dan variasi teritlak kuadratik (QGV). Manakala, parameter memori panjang iaitu parameter Hurst dianggar menggunakan beberapa kaedah heuristik dan kaedah semi-parametrik. Prosedur membangunkan model LMSV dan kaedah anggaran digunakan kepada harga indeks harian FTSE Bursa Malaysia KLCI untuk tempoh 20 tahun. Hasil kajian ini menunjukkan bahawa turun naik harga indeks mengalami memori panjang tetapi pulangan harga indeks tidak menunjukkan sifat-sifat keterusan yang kuat. Min ralat kuasa dua (RMSE) yang diperolehi daripada pelbagai kaedah menunjukkan bahawa prestasi model dan penganggar berupaya menerangkan pulangan harga indeks dengan baik.

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## LIST OF ABBREVIATIONS

|           |   |   |
|-----------|---|---|
| ACF       | - | Autocorrelation function                              |
| ARFIMA    | - | Autoregressive fractionally integrated moving average |
| CUSUM     | - | Cumulative sum  |
| $d$       | - | Fractional differencing parameter                     |
| DFA       | - | Detrended fluctuation analysis                        |
| fOU       | - | Fractional Ornstein-Uhlenbeck                         |
| fBm       | - | Fractional Brownian motion                            |
| fGn       | - | Fractional Gaussian noise                             |
| GPH       | - | Geweke Porter-Hudak test                              |
| $H$       | - | Hurst Parameter                                       |
| $H$ -sssi | - | Exponent self-similarity with stationary increments   |
| LMSV      | - | Long memory stochastic volatility                     |
| LSE       | - | Least square estimator                                |
| OLS       | - | Ordinary least squares                                |
| QGV       | - | Quadratic generalized variations                      |
| OML       | - | Quasi maximum likelihood                              |
| RMSE      | - | Root mean square error                                |
| R/S       | - | Rescaled range method                                 |
| SDE       | - | Stochastic differential equation                      |



## LIST OF SYMBOLS

|                               |   |                                       |
|-------------------------------|---|---------------------------------------|
| $\gamma(k)$                   | - | Autocovariance at time lag $k$        |
| $\rho(k)$                     | - | Autocorrelation at time lag $k$       |
| $f(\lambda)$                  | - | Spectral density                      |
| $\{X_t, t \in \mathbb{R}\}$   | - | Real-valued stochastic process        |
| $B_H(t)$                      | - | Fractional Brownian motion            |
| $\{Z_i^H, i \in \mathbb{Z}\}$ | - | Fractional Gaussian noise             |
| $\{\varepsilon_t\}$           | - | Gaussian white noise                  |
| $I_N(\lambda)$                | - | Periodogram                           |
| $N$                           | - | Number of samples                     |
| $(\Omega, \mathcal{F}, P)$    | - | Probability space                     |
| $\theta$                      | - | Model parameters                      |
| $\lambda$                     | - | Drift parameter                       |
| $\sigma_t$                    | - | Volatility Process                    |
| $\beta$                       | - | Diffusion coefficient                 |
| $S_t$                         | - | Index Prices                          |
| $\{X_t\}$                     | - | Returns                               |
| $\{X_t^2\}$                   | - | Squared returns                       |
| $ X_t $                       | - | Absolute returns                      |
| $(I_+^\alpha f)(t)$           | - | Riemann-Liouville fractional integral |
| $\Gamma(\cdot)$               | - | Gamma function                        |
| $\alpha$                      | - | Scaling exponent                      |
| $W_n^0(t)$                    | - | Empirical fluctuation process         |
| $\hat{\lambda}_N$             | - | Least square estimator                |

|                 |   |                                  |
|-----------------|---|----------------------------------|
| $V_{N,a}$       | - | Generalized quadratic variations |
| $\hat{H}_N$     | - | Estimator of $H$ using QGV       |
| $\hat{\beta}_N$ | - | Estimator of $\beta$ using QGV   |
| $p$             | - | Number of sample paths           |

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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Background of study**

Market indices are inter-related to the performance of local and global economies and become an important guideline of investor confidence. Therefore, people have always searched for the skill to predict behavior of prices in financial market. The very first success and famous mathematical option pricing model namely the Black-Scholes option pricing model which assumed the constant volatility had been proven to have many flaws (Casas and Gao, 2008; Chronopoulou and Viens, 2012b). The constant volatility assumption is inconsistent with the empirical observation of varying volatility across varying time. In fact, a typical financial time series of returns had many common properties or so-called “stylized facts”, such as excess kurtosis, volatility clustering and almost no serial correlation in the level but with a persistent correlation in the squared returns and absolute returns. This “stylized facts” phenomenon can be explained by an appropriate volatility model which will consider the persistence properties of the data set. Volatility is an essential factor in measurement of the variability in price movements. The volatility of the prices has significant influence on the dynamics of the financial time series. Thus, an appropriate model for volatility will help to improve the measurement and provide useful information to the investors and economist.

The aim of this thesis is to develop a procedure to determine the characteristic of the FTSE Bursa Malaysia KLCI index prices intensively and comprehensively, in both returns and volatility. The modeling and parameters estimation on the relationship between the returns and volatility intend to help investors to have a

clearer picture on the FTSE Bursa Malaysia KLCI index prices on decision making on their investments in Malaysia stock market. Besides that, the data analysis on the index prices can provide useful information to the Malaysia's government for successful development and implementation of policies on financial issues to improve our country's economy.

Constant volatility models have been proven in giving a poor fit on financial time series but the dynamic structures present a more realistic approach to volatility modeling. This is because volatility is affected by unpredictable changes such as the performance of the industry, political stability of particular country, news about new technology, natural disaster, product recalls and lawsuits that shall have positive and negative impact to the relevant company stocks, and therefore, the prices of the stock of a company are affected. Hence, many researches had been done in modeling volatility models in order to determine the dynamic fluctuation in the stock market. Among various volatility models, the autoregressive conditional heteroscedasticity (ARCH) model by Engle (1982) and generalised ARCH (GARCH) model by Bollerslev (1986) are very well known. The GARCH model assumes an ARMA-type structure for the volatility where the conditional volatility is a deterministic function of past returns. However, this assumption might be too restrictive in some of the problem and situations (Xie, 2008). As an example, the ARCH assumes that positive and negative shocks have same effects on volatility because it depends on the square of the previous shocks. In practice, it is well-known that price of a financial asset responds differently to positive and negative shocks. Therefore, another type of model, the stochastic volatility model which assumed to follow an autonomous and latent stochastic process will be more flexible.

Over the past two decades, many stochastic volatility (SV) models and estimation methods have been introduced to explain the market tendency. Stochastic volatility models have become popular for derivative pricing and hedging since the existence of a non-constant volatility surface has been classified. By assuming that the volatility of the underlying price is a stochastic process rather than a constant, it becomes possible to model derivatives more accurately. Stochastic volatility is a profound extension of the Black-Scholes model which describes a much more

realistic trend in the financial world. There are several popular stochastic volatility models such as the Heston model, Orstein-Unlenbeck model and Cox-Ingersoll-Ross (CIR) model. These models have been widely used in the field of mathematical finance to evaluate the stock market. Once a particular stochastic volatility model is chosen, the calibration against the existing market data need to be carried out in order to identify the most likely set of model parameters given the observed data.

The main assumption of the SV model is that the volatility is a log-normal process. Taylor (1986) and Hull and White (1987) were among the first to study the logarithm of the stochastic volatility as an Ornstein-Uhlenbeck process. The statistical properties and probabilistic of a log-normal are well known. However, parameter estimation is a very challenging task due to the difficulty in finding the maximum likelihood (ML) function. The sampling methods for estimating the stochastic volatility are generally based on Bayesian approach or classical approach. Examples of Bayesian approach are Gaussian mixture sampling, single site Sampler and multi-move sampler, whereas the classical methods include quasi maximum likelihood, simulated method of moments and importance sampling among others.

Recent studies had showed that some of the financial data exhibit the properties of long-range dependence. However, these properties cannot be captured by the ordinary stochastic models. Since the pioneer work on detection in the presence of the long range dependence or long memory in minimum annual flow series of the Nile River by Hurst (1951), numerous studies have been carried out for testing and modeling long memory in various areas. In general, autocorrelation function is a measure of the dependence or persistence between the previous state and current state at various lags in a time series. A process is considered to exhibit long-memory or long-term persistence dependence if there is a significant autocorrelation at long lags.

In empirical modeling of long memory processes, Granger and Joyeux (1980) was the first to introduce a new model based on ARCH-type namely fractional integrated autoregressive moving average (ARFIMA) which had greatly improved the applicability of long memory in statistical practice. The model is characterized by

hyperbolic decay rate of autocorrelation function. The term “fractional” is often used in the long memory context which usually refer to a model constructed using a generalized operation of non-integer order. In stochastic process, the models such as fractional Heston model and fractional Ornstein-Uhlenbeck (fOU) model had been modified to describe the long memory process in the sense that the present state of system is temporally dependent on all past states. Moreover, long memory is closely related with self-similar processes. Self-similar processes are stochastic models with the property that a scaling in time equivalent to an appropriate scaling in space. The connection between the two types scaling is determined by a constant which is known as Hurst exponent. Many of the empirical studies of long memory are based on the estimation method by Geweke and Porter-Hudak (1983). Besides that, there are some other estimation methods to detect the long memory based on the heuristic approaches such as rescaled range (R/S) statistics, detrended fluctuation analysis (DFA), periodogram method and aggregated variance method where neither of them needs any specific models assumptions.

The long memory in the volatility of the financial data had been discovered in the earlier of 1990's. Ding *et al.* (1993) were among the first to investigate that there is strong correlation between absolute returns of the daily S&P 500 index prices. The fractional power transformations of the absolute returns showed high autocorrelations for high lags which provide the evidence of long-range dependence (Crato and de Lima, 1994; Deo and Hurvich, 2001; Ezzat, 2013). Besides that, the long term correlation is also found in the squared returns on various financial markets (Casas, 2008; Xie, 2008; Günay, 2014).

Since the last decade, the issue of confusing long memory and occasional structural breaks in mean had received great attention (see, (Diebold and Inoue, 2001; Granger and Hyung, 2004; Smith, 2005; Cappelli and Angela, 2006; Yusof *et al.*, 2013; Mensi *et al.*, 2014). Diebold and Inoue (2001) showed that there is a bias in favor of finding long memory processes in a time series when structural breaks are not accounted. Indeed, there is evidence that a stationary short memory process that encounters occasional structural breaks in the mean may show a slower rate of decay in the autocorrelation function and other properties of fractionally

integrated processes (Cappelli and Angela, 2006). Thus, a time series with structural breaks can generate a strong persistence in the autocorrelation function which performs as the behaviour of a long memory process.

The early study of SV models was mainly focused on short memory volatility process. The long memory stochastic volatility (LMSV) model which is appropriate for describing series of financial returns at equally-spaced intervals of time had received extensive attention for last few years. Breidt *et al.* (1998) and Harvey (1998), simultaneously, was among the first who suggested a long memory stochastic volatility (LMSV) in discrete time where the log-volatility is modeled as an autoregressive fractional integrated moving average (ARFIMA) process. Comte and Renault (1998) proposed a continuous time fractional stochastic volatility model which adopted the fractional Brownian motion to replace the Brownian motion.

The LMSV models carry on many advantages of a general stochastic volatility model. However, unlike the usual short memory models, the LMSV model is neither a Markovian process nor can it be easily transformed into a Markovian process. This makes the likelihood evaluation and the parameter estimation for the LMSV model challenging tasks. Most of the previous research of LMSV model focused on the discrete time model which may due to the difficulty in constructing the computational work in continuous time model. In fact, the stock and index price process are only observed in discrete time, and the volatility itself cannot be directly observed, whether the underlying model is in discrete or continuous time or whether one believes that the underlying phenomena are discrete or continuous (Chronopoulou and Viens, 2012b). Therefore, from a financial modeling point of view, the statistical inference problem of estimating volatility under these conditions is then very crucial.

In terms of discrete-time models, Geweke and Porter-Hudak (1983) proposed a log-periodogram regression method which is also known as GPH method. In addition, Deo and Hurvich (2001) had presented the expressions for asymptotic bias and variance of the GPH estimators. Arteché (2004) proposed the Gaussian semiparametric or local Whittle estimator to estimate the long memory parameter.



There are very few papers that have developed the parameter estimations for long-memory stochastic volatility in continuous-time. Comte and Renault (1998) propose a discretization procedure to approximate the solution of their continuous-time fractional stochastic volatility (FSV) model and applied the log-periodogram regression approach to estimate the long memory parameter. Casas and Gao (2008) proposed the Whittle estimation method to estimate the parameters in a special class of FSV models. Chronopoulou and Viens (2012(a), 2012(b)) compared the performance of several long memory estimators and the implied value of  $H$  using real data of S&P 500 by calibrating it to option price.

Perhaps the most well-known approach of modelling long memory in continuous time stochastic volatility is to employ the fractional Brownian motion (fBm) as a long-memory driving source. The fractional Ornstein-Uhlenbeck (fOU) process is one of the popular model that contain the properties of long memory (see, (Cheridito *et al.*, 2003)). It would be optimal to estimate the parameters of fractional Ornstein-Uhlenbeck process and the long memory parameter jointly. But, most of the long memory models, still none provides a rigorous way for estimating a joint vector of parameters. Chronopoulou and Viens (2012(a), 2012(b)) suggested estimating the parameters separately and proposed to use calibration technique to fit models with various Hurst parameters. Many authors estimated the drift parameter and diffusion parameter separately with assumption of the Hurst parameter is known (see, (Hu and Nualart, 2010; Xiao *et al.*, 2011; Brouste and Iacus, 2013; Wang and Zhang, 2014)).

## 1.2 Statement of the Problem

One of the perplexing issues with regards to the detection of long memory is the confusion between long memory and non-linear effects such as parameter changes in time. There is evidence that a stationary short memory process that has occasional structural breaks in the mean can show a slower rate of decay in the autocorrelation function and other properties of long memory process. The structural change in the series camouflages the stationary short memory process. The long

memory may be apparent for a certain sample, but a deeper investigation would be needed to show the long memory property as results from structural breaks or slow regime switching in the time series. Thus, long memory may be detected spuriously if structural break are not accounted for. Many authors had found that the financial time series exhibit the long memory properties. The question to ask is if it is really a long memory process or a short memory with structural break? One of our main concerns in this study is to investigate whether our financial data set exhibits a true long memory process.

Discovering the behaviour of market prices is not a simple task. The stock market prices tend to have complicated distributions with strong skewness and fat tails which commonly known as “stylized facts”. It is very essential to estimate the volatility in order to forecast the dynamic of the prices, i.e. what is the expected prices for tomorrow and how much it will differ from today’s price. Therefore, an appropriate modelling for the volatility becomes our second task in our study. There are a lot of evidences showing the existence of strong persistence in volatility of financial series. This study tends to propose a stochastic volatility model that will take into consideration of the volatility persistence.

The estimation of the volatility process is one of the most difficult and complicated problems in econometrics. There are neither volatility simulation techniques nor volatility data collections are completely ideal. The main difficulties are the fact that volatility itself is never directly observed. Therefore, in practice, one would be restricted to use the values of asset at discrete time even for the most liquid indexes or assets. Thus, we tend to propose a procedure to estimate the volatility process of the financial data series.

### **1.3 Objectives of the Study**

The objectives of the study are:

1. To identify the true long memory from long memory process on the returns and volatility of the financial time series.
2. To propose continuous-time diffusion process in state space form for more flexible modeling of continuous dynamics in financial time series.
3. To establish a structured procedure in estimating the volatility process from the proxies of volatilities with the parameters estimation on the long memory stochastic volatility model.
4. To assess the long memory stochastic volatility model and estimation methods.

#### **1.4 Scope of the Study**

The scope of this research is given as follows:

1. This study establishes a general framework for analysing the closing index prices of FTSE Bursa Malaysia KLCI beginning from 3<sup>rd</sup> December 1993 until 31<sup>st</sup> December 2013. Here, the long memory properties of the closing index prices will be determined based on the returns and the volatilities of the data. The structural break analysis will be carried out to justify whether it is the true long memory or the spurious one.
2. This work presents the LMSV model with explanation on its basic properties. The LMSV state space model with fractional Ornstein-Uhlenbeck process including the long memory properties will be constructed. The models are developed to observe the volatilities persistence on the index prices.
3. The long memory parameter will be estimated with the Heuristic and semi-parametric methods. Whereas, the drift parameter and the diffusion coefficient will be estimated using least square method and quadratic generalised variation method respectively. This study performs the

numerical simulation of the model based on Monte Carlo method to illustrate the performance of the model and estimations methods.

4. The complete procedures are developed to analyse the characteristic of the closing index prices and constructing the LMSV model with estimation methods which will be assessed by their consistency. The comparison between the simulated and empirical returns is evaluated by the root mean square error and descriptive statistic.

## **1.5 Significance of the Study**

The contribution of this research is in developing the procedures to analyse the index prices of FTSE Bursa Malaysia KLCI. As we all know, the volatility which measures the variability in price movement is at the centre of models for financial time series. This research considers a general class of stochastic volatility models either with long range dependence, intermediate range dependence or short range of dependence.

Long memory is one of the characteristic in financial time series. The return of the prices usually exhibits little or no autocorrelation, but volatility often has a strong autocorrelation structure. However, an argument exists saying that a short memory process with an occasional structural break can show the properties of long memory process. A spurious long memory will be detected if the structural break of the time series is not considered. Therefore, this study develops a strategy to detect true long memory of the returns and volatilities in financial data set.

Secondly, we propose a long memory stochastic volatility model in state space form using fractional Ornstein-Uhlenbeck process that can capture the characteristic observed in the financial time series. The LMSV model is more flexible assuming the volatility follows an autonomous and latent stochastic process. This model can provide a useful way of modeling the relationship between the

returns and the volatility of the series exhibiting strong persistence in its level yet with varying time.

The LMSV model can help to identify the structure of the index prices in deriving the returns and volatility patterns. The estimated parameters on drift, diffusion coefficient and Hurst parameter in the model are practically useful for investor to have a clear picture on characteristics of the index prices. The goals of this research are to study the properties of FTSE Bursa Malaysia KLCI index prices via the long memory stochastic volatility models to solve the problem of excessive persistence in the composite linear and nonlinear models by introducing a probabilistic approach in allowing different volatility states in time series. The estimated returns show good result of the model in describing the dynamics of the FTSE Bursa Malaysia KLCI index prices. In this way, the accurate information can be provided for the forecasting in the future.

To the best of the author's knowledge, there are no studies on applying long memory stochastic volatility models with parameters estimation to address the issues of the Malaysia economics. The proposed methods on estimating the volatility process from the proxies of volatilities using the modified LMSV model are the main contribution of this research. By establishing a complete procedure to identify the characteristic of the FTSE Bursa Malaysia KLCI index prices, the modified LMSV model manage to explain the Malaysia market tendency.

## **1.6 Organization of Thesis**

The structure of this thesis can be summarized as follows. The thesis consists of introductory material which includes motivation, objectives, scope and significance of study in Chapter 1, literature review of the LMSV model and estimations methods in Chapter 2, the methodology and our novel contributions in Chapter 3, 4, 5, and the conclusion and future works in Chapter 6.

Chapter 2 presents literature review on long memory process with stochastic volatility and the estimations methods. The basic mathematical formulation for long memory process and stochastic volatility are defined. Then, the previous studies on LMSV models are introduced. The fractional Ornstein-Uhlenbeck model and its parameters estimation methods are also discussed. Furthermore, the literature on the methods of estimating the long memory parameter based on heristic and semi-parametric approaches are presented. The application of the long memory in financial time series will be evaluated from the point of view of modelling and the methods employed to estimate the parameters. Last but not least, the challenges of parameters estimation of LMSV model are included.

The long memory processes with definition and several aspects of their characteristics are introduced in Chapter 3. The long memory process will be defined in terms of its autocorrelations and spectral density. Then, the self-similar processes of long memory and fractional Brownian motion are described. Here, the modeling of long memory process in discrete time namely autoregressive fractional integrated moving average (ARFIMA) model and continuous time namely fractional Ornstein-Uhlenbeck (fOU) model are also discussed briefly. In this study, the focus mainly on fOU model in the modeling of long memory stochastic volatility model. The fractional calculus that is applied in this study will be explained as well.

In Chapter 4, a discussion on the modeling and parameters estimation of long memory stochastic volatility is given. First of all, the direction of volatility modeling in financial market is discussed. Then, some of the basic stochastic volatility model based on discrete and continuous time is presented. Next, this research modified a long memory stochastic volatility (LMSV) model in state space form. Methods for testing the existence of long memory are illustrated here. Moreover, the method for structural break analysis is studied in order to determine the true long memory. While, methods of parameters estimation included the drift and diffusion coefficient in the fOU volatility process of the LMSV model are also discussed. Last but not least, the methods to show the efficiency of the model and parameters estimation methods will be derived.

Chapter 5 presents the analysis of the index prices of FTSE Bursa Malaysia KLCI based on the proposed model and estimation methods of this research. Firstly, the description of the data and its transformation is presented. Then, how the long memory parameter estimation using heuristic and semiparametric approaches are employed on the data is shown. The results are given based on time domain and frequency domain according to the methods. Further, the results of structural break analysis are also presented. This is the chapter where the parameters estimation on the fractional Ornstein-Uhlenbeck (fOU) model for the long memory stochastic volatility is carried out based on the methodologies. Lastly, the contribution in this study is highlighted.

Chapter 6 is the final chapter which summaries the research findings. Besides, some suggestions for future works which might be potential and useful for further development or improvement of the proposed models and estimation methods are discussed.

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